Multi-Activity, Multi-Day Shift Scheduling Problem Definition

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The planning horizon spans exactly h consecutive periods of 24 hours. The horizon starts and finishes at 06:00. D represents the set of calendar days covered by the planning horizon, where a calendar day is the period from one midnight to the next midnight. Note that $|D|=h+1$ because the horizon starts and ends at a time of 06:00, so it covers $h+1$ calendar days. The planning horizon is split into 15-minute intervals. Therefore, an $h=7$ horizon has 672 intervals. S_d represents the possible shift start times on each calendar day d. The start times correspond to interval indexes from I. Shifts must start at 00:00 or 06:00-10:00 or 14:00-18:00 or 20:00-23:45. A shift cannot start at 00:00 on the first day because it is outside of the planning horizon. The only start time on the last day that is within the planning horizon is 00:00. The problem has a continuous horizon meaning that a shift which starts on day d can span midnight and finish on $d+1$. L represents the possible shift lengths. The minimum shift duration is six hours and the maximum duration is ten hours. The shift lengths are in 15-minute intervals i.e. $\{24\ldots40\}$. If there is no shift assigned on a day then it is assigned the length zero.

Parameters:

A	set of activities.
h	number of consecutive periods of 24 hours in the planning horizon.
D	set of consecutive calendar days covered by the planning horizon, where a calendar day is the period from one midnight to the next midnight = $\{0\ldots h\}$.
E	set of employees.
I	set of intervals in the planning horizon = $\{0\ldots(h*96)-1\}$
L	set of possible shift lengths = $\{0, 24\ldots40\}$.
S_d	set of possible shift start times on each day d.
b_e	minimum number of minutes work that employee e must be assigned during the horizon.
c_e	maximum number of minutes work that employee e can be assigned during the horizon.
v_{ia}	minimum total number of employees required for activity a at interval i.
w_{ia}	maximum total number of employees required for activity a at interval i.

Decision variables:

$p_{eds} \in \{0,1\}$	1 if employee e's shift on day d starts at time s, 0 otherwise
$q_{edl} \in \{0,1\}$	1 if employee e's shift on day d has length l, 0 otherwise
$x_{eia} \in \{0,1\}$	1 if employee e is assigned activity a at interval i, 0 otherwise
$y_{ia} \in \mathbb{Z}^*$	total employees above the max demand for activity a at interval i.

Constraints:

1. An employee can only have one shift each day and each shift can only have one start time and one length.

$$\sum_{s \in S_d} p_{eds} \leq 1, \quad \forall \mathrm{e} \in \mathrm{E}, d \in D$$

$$\sum_{l \in L} q_{edl} \leq 1, \quad \forall \mathrm{e} \in \mathrm{E}, d \in D$$

Using the binary variables *p* and *q*, we now define the following expressions:

The start time, *t*, in intervals for employee *e*'s shift on day *d*

$$t_{ed} = \sum_{s \in S_d} p_{eds} * s, \quad \forall \mathrm{e} \in \mathrm{E}, d \in D$$

The duration, *n*, in intervals for employee *e*'s shift on day *d*

$$n_{ed} = \sum_{l \in L} q_{edl} * l, \quad \forall \mathrm{e} \in \mathrm{E}, d \in D$$

The end time, *u*, in intervals for employee *e*'s shift on day *d*

$$u_{ed} = t_{ed} + n_{ed} \qquad \forall \mathrm{e} \in \mathrm{E}, d \in D$$

A binary indicator, *z*, to indicate if an employee *e* has a shift on day *d* or not

$$z_{ed} \Leftrightarrow n_{ed} > 0, \qquad \forall \mathrm{e} \in \mathrm{E}, d \in D$$

2. Shifts must finish within the planning horizon. This constraint is only required on the last two days in the planning period because the maximum shift lengths prevent this constraint being needed on earlier days.

$$u_{ed} \leq h * 96, \quad \forall e \in E, d \in \{h-1 \ldots h\}$$

3. There must be at least 14 hours rest after a shift.

$$z_{e(d+1)} \Rightarrow u_{ed} \leq t_{e(d+1)} - (14 * 4), \quad \forall e \in E, d \in \{0 \ldots h-1\}$$

4. Minimum and maximum work time. The total work time during the horizon assigned to each employee must be within a minimum and a maximum number of minutes specified for each employee. Note that it would be simple to extend the problem here, if needed, by specifying the requirements per day or per week and/or per activity.

$$b_e \leq \sum_{d \in D} n_{ed} * 15 \leq c_e, \quad \forall e \in E$$

5. Maximum five consecutive shifts. The maximum number of shifts an employee can work without a day off is five. A day is considered as a working day if a shift starts on that day.

$$\sum_{j=d}^{d+5} z_{ed} \leq 5, \quad \forall e \in E, d \in \{0 \ldots h-5\}$$

6. Minimum activity duration. Employees must be assigned to an activity for a minimum of one hour. This means an employee cannot switch to a different activity until they have worked at least one hour on the activity.

$$x_{eia} + \left(s - \sum_{j=i+1}^{i+s} x_{eja} \right) + x_{e(i+s+1)a} > 0,$$

$$\forall e \in E, a \in A, s \in \{1...3\}, i \in \{0...h * 96\}$$

7. Activities can only be assigned between shift start and end times, and an employee cannot have more than one activity during an interval. Because the planning horizon is continuous, meaning that shifts can span midnight, there are two cases to consider: day zero and all other days.

For day zero

$$\sum_{a \in A} x_{eia} \Leftrightarrow t_{e0} \leq i < u_{e0}, \qquad \forall e \in E, i \in \{0 \ldots 95\}$$

For day > 0

$$\sum_{a \in A} x_{eia} \Leftrightarrow t_{e(d-1)} \leq i < u_{e(d-1)} \vee t_{ed} \leq i < u_{ed}$$

$$\forall e \in E, d \in \{1..h\}, i \in \{d * 96...(d+1) * 96 - 1\}$$

8. Minimum activity cover demand requirements.

$$v_{ia} \leq \sum_{e \in E} x_{eia} - y_{ia} \leq w_{ia}, \quad \forall i \in I, a \in A$$

Objective function:

$$Minimise \sum_{i \in I} \sum_{a \in A} y_{ia} y_{ia}$$

The objective function models a requirement to minimise over staffing. The variable y_{ia} is the total number of assigned staff above the preferred maximum cover level for each activity *a* on each interval *i*. Note that the objective function is quadratic, meaning that this is a Mixed Integer Quadratic Program (MIQP).